

been obtained exhibiting the expansion of the liquid, the pressure of the vapour, and the compressibility of the substance in the gaseous and liquid conditions; and from these results, the densities of the saturated vapour and the heats of vaporisation have been deduced. The temperature range of these observations is from -18° to 223° C.

It is the authors' intention to consider in full the relations of the properties of alcohol and ether; in the meantime it may be stated that the saturated vapour of ether, like that of alcohol, possesses an abnormal density, increasing with rise of temperature and corresponding rise of pressure; that at 0° the vapour-density is still abnormal, but appears to be approaching a normal state; and that the apparent critical temperature of ether is 194.0° C.; the critical pressure very nearly 27,060 mm. = 35.61 atmospheres; and the volume of 1 gram of the substance at 184° between 3.60 and 4 c.c.

III. "On the Working of the Harmonic Analyser at the Meteorological Office." By ROBERT H. SCOTT, F.R.S., and RICHARD H. CURTIS, F.R. Met. Soc. Received May 6, 1886.

On the 9th of May, 1878, Sir W. Thomson exhibited to the Society a model of an integrating machine, which consisted of a series of five of the disk, globe, and cylinder integrators, which had been devised two years earlier by his brother Prof. James Thomson, and a description of which will be found in the "Proceedings of the Royal Society," vol. xxiv, p. 262. Sir W. Thomson's paper describing this model will be found in vol. xxvii of the "Proceedings," p. 371; and reference should be made to both these papers for an explanation of the principle of the machine. In the communication last named it is stated that the machine was about to be "handed over to the Meteorological Office, to be brought immediately into practical work."

The model was received at the Office in the course of the month, and was at once set in action; the results of the preliminary trials, when obtained, being referred to a Committee consisting of the late Prof. H. J. S. Smith and Prof. Stokes, who, on the 5th of July following, submitted to the Meteorological Council a favourable report on the performance of the model.

The Council at once resolved to have a machine constructed, which should be specially adapted to the requirements of the work for which it was intended, viz., the analysis of photographic thermograms and barograms.

In preparing a working design for actual execution, it was found necessary to make several modifications in the details of the mechanical arrangements of Sir W. Thomson's original model, and these were

mainly worked out by Prof. Stokes and Mr. de la Rue. Plans were obtained from two firms of mechanical engineers, and those of Mr. Munro being ultimately adopted, the construction of the instrument was entrusted to him. It was considered sufficient to limit the action of the machine so as to extend only to the determination of the mean, and the coefficients as far as those of the third order, in the expression

$$E = a + a_1 \cos \theta + b_1 \sin \theta + a_2 \cos 2\theta + b_2 \sin 2\theta + a_3 \cos 3\theta \\ + b_3 \sin 3\theta + \&c.,$$

and to obtain these it was necessary to have seven sets of spheres, disks, and cylinders.

A description of the machine, as actually constructed, was published in "Engineering" for December 17th, 1880, and we are indebted to the proprietors of that journal for permission to reproduce the engravings which illustrate that description, as well as a portion of the text, which we now proceed to quote:—

"The machine is shown in the accompanying engravings, figs. 1 to 3; figs. 2 and 3 showing details of the arrangements of the ball, disk, and cylinder. In principle it is, of course, precisely similar to its predecessor—differing from it only in constructive details intended to secure stability, and accuracy in its movements. Instead of being largely made of wood, as was the case with the model, it is entirely of metal; the cast-iron frames carrying the disks being secured to a firm iron bed supported by two substantial uprights; the disks themselves are of gun-metal, and the spheres of steel carefully turned, and nickel-plated to prevent rusting; the horizontal bar carrying the forks for moving the spheres is also of steel and plated, and is carried above the disks upon five iron uprights or guides. The forks are provided with adjusting screws allowing of very accurate centering of the spheres upon the disks, and adjusting screws are likewise provided for the frames carrying the recording cylinders, by which their parallelism to the faces of the disks can be rigidly secured. The spheres are not touched by the forks themselves, but by the flat faces of two screws passing through their lower extremities, and in this way a ready means of preventing looseness or tightness of the spheres in the forks is provided.

"Each recording cylinder has attached to it a counter for registering the number of its complete revolutions, and to secure a maximum of freedom in their movements the spindles of the cylinders, as well as the slides carrying the racks for giving motion to the disks, and the horizontal steel bar, are all made to run upon friction rollers; the slides have in addition counterpoises attached to them to prevent error from backlash.

"The motion of the shaft at the rear of the machine is communi-

FIG. 1.

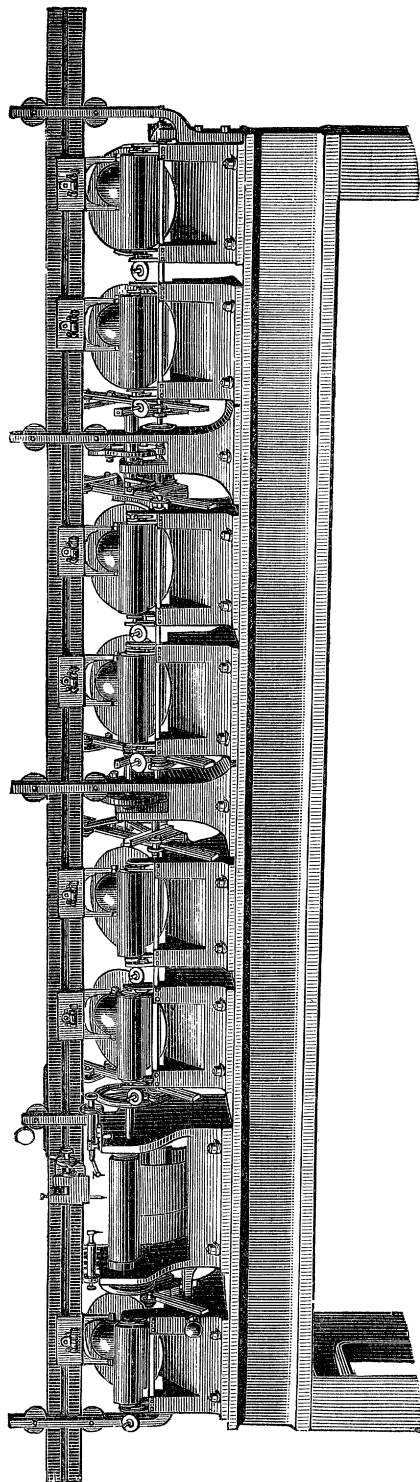


FIG. 2.

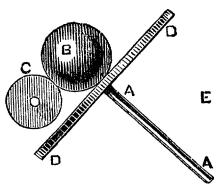
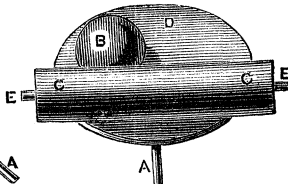


FIG. 3.



ected to the second and third pairs of cranks through sets of toothed wheels, so arranged that they may, if desired, be changed for others of different ratios to the cylinder carrying the curves, and in this way the terms of other orders of the expansion may be obtained, should they be required, with the same instrument, merely going over the curves afresh, and using wheels of the proper ratios in place of those used for the first, second, and third pairs of terms.

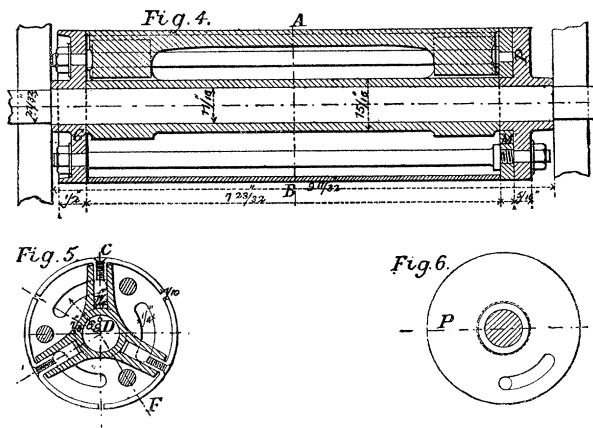
"The cylinder over which the curves have to pass is provided with an ingenious arrangement by which it can be expanded or contracted to alter its circumferential measurement by about four-tenths of an inch, so that within certain limits variations in the length of the time ordinates of the curves can be very readily allowed for. To effect this the cylinder (see figs. 4, 5, and 6) is made in three sections, each provided with an eccentric movement; of course, except when these are at their normal positions the "cylinder" is not truly cylindrical, but still, even when moved to their extreme limits, the deviation is not so large as to cause any inconvenience in its use. Adjustments are also supplied to the pointers, of which two are used at once, the one to follow the outline of the curve dealt with, and the other the zero line from which the ordinates are measured; and throughout the machine all racks and toothed wheels are skew cut to further lessen the risk of error from backlash.

"The height of the machine is 3 feet 8 inches, and the length of the steel bar, which is rather longer than the bed of the machine, 9 feet.

"The machine has now for some time been in regular use at the Meteorological Office, and notwithstanding the weight and solidity of some of its parts, the whole is so nicely balanced and fitted that it works with the utmost ease and smoothness."

The machine was delivered at the Office in December, 1879, and a lengthened series of trials was at once commenced, to determine its constants, and thoroughly test the accuracy of its working, for which purpose systems of straight lines and curves, of which the values were known, were first used. A few small unforeseen difficulties were early met with, necessitating slight modifications in some portions of the instrument.

The chief of these faults was a slight turning of the cylinders upon their axes, when the balls were moved to and fro along the disks,



parallel to the axes of the cylinders. The movement was always in the same direction, namely, towards the disks, whether the ball was moved to the right or left. After the trial of many expedients the defect was finally, in great measure, overcome by attaching weights to the spindles of the cylinders. It however still exists in the machine to a slight extent, and its effect is to *decrease* the readings on the cylinders by a very small amount.

It was decided to employ the analyser, in the first instance, in the determination of temperature constants, and careful comparisons have been made of the results obtained by its means, with those got by actual measurement of the photographs and numerical calculations, as will presently be mentioned, and the accordance is so very close as to prove that the machine may safely be trusted to effect reductions which could only otherwise be accomplished by the far more laborious process of measurement and calculation.

It will facilitate an apprehension of the method of using the machine to give a somewhat detailed account of the operations involved in the treatment of the curves, with an example of the manner in which the readings of the machine are recorded and dealt with.

The machine is furnished with three pairs of recording cylinders and disks, numbered consecutively 1 to 6, which give the coefficients for the first three pairs of terms of the expansion, and in addition a seventh cylinder and disk from which the mean is obtained. In the thermograms which supply continuous photographic records of the march of temperature, the trace for twenty-four hours covers a length of 8.75 inches, while a vertical height of about 0.7 inch* corresponds to a range of ten degrees in temperature; each thermograph sheet contains the record for forty-eight hours.

* This value varies slightly for each observatory.

Conveniently placed in the machine is a cylinder or drum, the circumference of which is equal to the length of twenty-four hours upon the thermograms. Round this cylinder the thermograms are rolled, the fluctuations of temperature indicated by the curves being followed, as the cylinder revolves, by a combination of the movement of the cylinder with that of a pointer moving in a line parallel to its axis.

The handle by which the cylinder is turned gives motion at the same time to the seven disks of the machine, and the operator thus controls by his left hand both the speed with which the curves are paid through the machine and the consequent velocity of the angular motion of the disks, while by a suitable contrivance, the movements of the pointer, governed by his right hand and following the curve, produce on the face of the disks corresponding movements to the right or left of the balls by which the motion of the disks is conveyed to the recording cylinders.

At the commencement of an operation all the cylinders are set to zero; the twelve months curves are then passed consecutively through the instrument; the first pair of cylinders, which gives the coefficients of the first order, and also the mean cylinder, 7, being read for each day, while cylinders 3 and 4, and 5 and 6, which give the coefficients of the second and third orders respectively, are only read for each five days and at the end of each calendar month. The numbers on the cylinders are, however, progressive, so that the increments upon them for any given period could very easily be obtained. The form in which the readings are recorded is as follows:—

Readings of the Recording Cylinders of the Harmonic Analyser.

Dry-bulb Thermometer Curves, from July 30 to August 3, 1882.

Kew Observatory.

Month and day.	First order.			
	Cylinder 1.		Cylinder 2.	
	Reading at midnight.	Difference from last reading.	Reading at midnight.	Difference from last reading.
June 30	+ 10·480	—	— 12·333	—
July 30	+ 12·540	+ 0·103	— 14·926	— 0·118
„ 31	+ 12·678	+ 0·138	— 15·004	— 0·078
August 1	+ 12·773	+ 0·095	— 15·069	— 0·065
„ 2	+ 12·814	+ 0·041	— 15·207	— 0·138
„ 3	+ 12·897	+ 0·083	— 15·287	— 0·080

Month and day.	Second order.			
	Cylinder 3.		Cylinder 4.	
	Reading at midnight.	Difference from last reading.	Reading at midnight.	Difference from last reading.
June 30.....	-3·953	—	-3·256	—
July 30.....	—	—	—	—
„ 31.....	-4·054	±0·000	-3·454	-0·080
August 1.....	—	—	—	—
„ 2.....	—	—	—	—
„ 3.....	-4·084	-0·030	-3·466	-0·012

Month and day.	Third order.			
	Cylinder 5.		Cylinder 6.	
	Reading at midnight.	Difference from last reading.	Reading at midnight.	Difference from last reading.
June 30.....	-1·877	—	-1·827	—
July 30.....	—	—	—	—
„ 31.....	-2·674	-0·031	-2·391	-0·032
August 1.....	—	—	—	—
„ 2.....	—	—	—	—
„ 3.....	-2·786	-0·112	-2·456	-0·065

Month and day.	Mean.		Midnight reading of curve.	Difference from last reading.
	Cylinder 7.			
	Reading at midnight.	Difference from last reading.		
June 30.....	- 2·285	—	—	—
July 30.....	+ 52·637	+ 2·302	—	—
„ 31.....	+ 54·554	+ 1·917	—	—
August 1.	+ 57·177	+ 2·623	—	—
„ 2.....	+ 59·753	+ 2·576	—	—
„ 3.....	+ 61·299	+ 1·546	—	—

At present only the monthly increments of the readings have been dealt with, so as to obtain the coefficients of the mean daily variation for each month of the year. The process followed is, therefore, simply to divide the monthly increment by the number of days in the month, and then to multiply the quotient by a factor which is determined by the scale-value of the thermograms, and which will therefore be different for each observatory.

The ratios of the factors for cylinders 1 to 6 to that of No. 7 were very carefully determined from a series of experimental curves, of which the values were known. The numerical factor is obtained for each observatory by obtaining on cylinder No. 7 the scale reading corresponding to a vertical movement of the pointer of 10° on the thermogram, which in the case of Kew is 0.75 inch. The factor for cylinders Nos. 1 and 2 is eight times that for cylinder No. 7; the factor for Nos. 3 and 4 is four times that quantity, and for Nos. 5 and 6 is eight-thirds of that quantity. The signs of the factors depend on the direction in which the disks and cylinders are caused to revolve. The constant quantity added to the reduced reading of cylinder No. 7 corresponds to the temperature which is assumed as the zero at the commencement of the operation.

As an illustration, the case of Kew for July, 1882, may be taken, the final readings of the cylinders for which month are above given. The increments for the month shown by these figures are as follows:—

Cylinder	1.	2.	3.	4.	5.	6.	7.
Observed increment	+2.198	-2.671	-0.101	-0.198	-0.797	-0.564	+56.839
Divided by 31 (the number of days)	+0.071	-0.086	-0.003	-0.006	-0.026	-0.018	+1.834
Factor	-53.52	+53.52	-26.76	-26.76	-17.84	-17.84	+6.69
Coefficient deduced	-3.80	-4.60	+0.08	+0.16	+0.46	+0.32	+12.27
Add constant.....							48.17
Mean temperature							60.44

After some trials with the curves for the year 1871, the year 1876 was taken up, inasmuch as for that year the records had been discussed by Mr. H. S. Eaton, M.A., F.R. Met. Soc., who had calculated the hourly means of the various meteorological elements for each month separately, and who kindly placed his results at the disposal of the Council.

The working of the machine was thus subjected to an exact test by comparing the results obtained by it with the coefficients in the harmonic series which were calculated from Mr. Eaton's means; and their trustworthy character, and the adequacy of these calculations to

serve as a standard with which the coefficients obtained by means of the machine might be compared, was established by calculating them from the *odd* and *even* hours, quite independently, for all the seven observatories.

The outcome of this experiment was thoroughly satisfactory, and the entire series of results obtained both by calculation and from the machine was published as Appendix IV to the Quarterly Weather Report for 1876, together with a Report prepared by Prof. Stokes, the concluding paragraphs of which may be quoted here, since they sum up in a few words the conclusions arrived at.

"Disregarding now the systematic character of some of the errors, and treating them as purely casual, we get as the average difference between the constants as got by the machine and by calculation from the twenty-four hourly means 0.065° . It may be noticed, however, that the numbers are unusually large (and at the same time very decidedly systematic) in the case of the second cylinder of the first order (b_1), for which the average is as much as 0.150° , the seventh of a degree.

"If b_1 be omitted, the average for the remaining cylinders of the machine is reduced to 0.047° .

"We see, therefore, that with the exception perhaps of b_1 , the constants got by the machine for the mean of the days constituting the month are as accurate as those got by calculation, which requires considerably more time, inasmuch as the hourly lines have to be drawn on the photograms, then measured, then meaned, and the constants deduced from the means by a numerical process by no means very short."

The curves for the twelve years 1871 to 1882 inclusive have now been passed through the machine, and the results obtained have been carefully checked so far as the arithmetical work involved is concerned, upon a plan approved by the Council. No *direct* check, short of passing the curves a second time through the machine can however at present be put on any portion of the results except as regards the means, which have been compared with the means calculated from the hourly readings obtained by measurement from the curves. The results of this work will be published as an appendix to the "Hourly Readings from the Self-Recording Instruments," for 1883, but the general results may here be stated.

As a rule, the monthly means yielded by the harmonic analyser agree well within a tenth of a degree with those obtained by calculation from the hourly measurements of the curves; and although in some exceptional cases larger differences have been found, amounting in rare instances to as much as half a degree, it is probable that generally these are less due to defects in the working of the instrument than to other causes. In some cases large breaks in the curves, due

to failure of photography, &c., were interpolated when the curves were passed through the machine, but not when the means were worked out from the hourly measurements. Some differences rather larger than usual, and confined chiefly to the earliest years dealt with, have been ascertained to have arisen from the circumstance that when the curves were first measured, to obtain hourly values, the method of making the measurements was not the same as that found by subsequent experience to be the preferable; and also that in some cases the scale-values first used were less accurately determined than has since been found possible.

In both these respects the two methods were on a par in the later years dealt with, and therefore the fairest comparison is to be had with their means.

For 1880, the average difference of the monthly mean for all the seven observatories is 0.09° ; for 1881 it is 0.05° ; and for 1882 0.06° ; and in these three years a difference of 0.3° between the analyser and calculated means occurred but once, and of 0.2° but five times.

What has been said is sufficient to show that the instrument is completely applicable to the analysis of thermograms.

It has also been employed on the discussion of barograms, and the curves for the years 1871, 1872, and 1876 have been passed through the machine.

The year 1876 was selected owing to the existing facilities for comparing the resulting figures with those obtained by calculation from Mr. Eaton's means, and the result in this case was equally satisfactory with that for temperature already mentioned.

In conclusion, the Fellows may perhaps be reminded that on June 18th, 1874, one of us (Mr. Scott) read a paper "On the use of an Amsler's Planimeter for obtaining mean values from Photographic Curves," "Proc. Roy. Soc.," vol. 22, p. 435. This paper contains a table exhibiting the differences between the means so obtained and those yielded by the hourly values.

We reproduce this table, appending to it the values obtained from the analyser for the same period.

It will, of course, be remembered that the mean is the only result which can be got from the planimeter, while it is but a small part of what is yielded by the harmonic analyser; but a comparison of the figures obtained by the two instruments from the same photographic curves may be interesting, as being got in the case of the planimeter from an instrument in which there is a combined "rolling and slipping" movement, while the movement in the analyser is one of "pure rolling contact."

Comparison of Temperature Means obtained from Kew Photographic Thermograms, by Amsler's Planimeter, and Sir W. Thomson's Harmonic Analyser respectively, with those obtained by Numerical Calculation from Hourly Measurements of the Curves.

Groups of five days.	Means.			Differences.	
	Tabulations.	Planimeter.	Harmonic analyser.	T-P.	T-A.
1872. Apr. 1—5 ..	44·8	45·1	44·8	-0·3	0
6—10 ..	48·1	48·4	48·1	-0·3	0
11—15 ..	52·8	52·7	52·7	+0·1	+0·1
16—20 ..	43·9	44·2	44·1	-0·3	-0·2
21—25 ..	48·0	48·1	48·0	-0·1	0
26—30 ..	53·3	53·5	53·4	-0·2	-0·1
May 1—5 ..	53·5	53·6	53·5	-0·1	0
6—10 ..	48·1	48·5	48·2	-0·4	-0·1
11—15 ..	46·9	46·9	46·7	0	+0·2
16—20 ..	47·3	47·4	47·2	-0·1	+0·1
21—25 ..	50·8	50·9	50·6	-0·1	+0·2
26—30 ..	59·1	59·0	59·2	+0·1	-0·1
31—4 ..	53·4	53·5	53·4	-0·1	0
June 5—9 ..	53·8	54·0	53·9	-0·2	-0·1
10—14 ..	58·0	58·1	58·0	-0·1	0
15—19 ..	68·7	68·7	68·5	0	+0·2
20—24 ..	62·6	62·7	62·3	-0·1	+0·3
25—29 ..	59·5	59·7	59·5	-0·2	0
30—4 ..	62·6	62·7	62·6	-0·1	0
July 5—9 ..	66·0	66·3	66·1	-0·3	-0·1
10—14 ..	62·6	62·7	62·5	-0·1	+0·1
15—19 ..	61·0	61·0	61·1	0	-0·1
20—24 ..	69·7	69·8	69·7	-0·1	0
25—29 ..	70·1	70·2	70·1	-0·1	0
30—3 ..	59·3	59·4	59·4	-0·1	-0·1
Aug. 4—8 ..	59·9	60·0	60·0	-0·1	-0·1
9—13 ..	59·7	59·8	59·7	-0·1	0
14—18 ..	62·4	62·5	62·4	-0·1	0
19—23 ..	65·3	65·1	65·2	+0·2	+0·1
24—28 ..	60·6	60·6	60·5	0	+0·1
29—2 ..	59·7	59·6	59·7	+0·1	0
Sept. 3—7 ..	65·7	65·7	65·7	0	0
8—12 ..	62·6	62·7	62·7	-0·1	-0·1
13—17 ..	62·2	62·1	62·2	+0·1	0
18—22 ..	48·8	49·0	49·0	-0·2	-0·2
1873. Jan. 31—4 ..	32·9	33·2	33·0	-0·3	-0·1
Feb. 5—9 ..	34·7	34·9	34·8	-0·2	-0·1
10—14 ..	37·2	37·3	37·1	-0·1	+0·1
15—19 ..	36·4	36·7	36·5	-0·3	-0·1
20—24 ..	32·9	33·2	32·9	-0·3	0
25—1 ..	39·7	39·8	39·7	-0·1	0
Mar. 2—6 ..	44·3	44·4	44·4	-0·1	-0·1
7—11 ..	42·3	42·5	42·4	-0·2	-0·1
12—16 ..	37·4	37·5	37·4	-0·1	0
17—21 ..	39·5	39·7	39·6	-0·2	-0·1
22—26 ..	43·6	43·8	43·7	-0·2	-0·1
27—31 ..	47·3	47·4	47·3	-0·1	0
Mean difference				-0·12	-0·01
Mean difference from mean				0·07	0·07

FIG. 1.

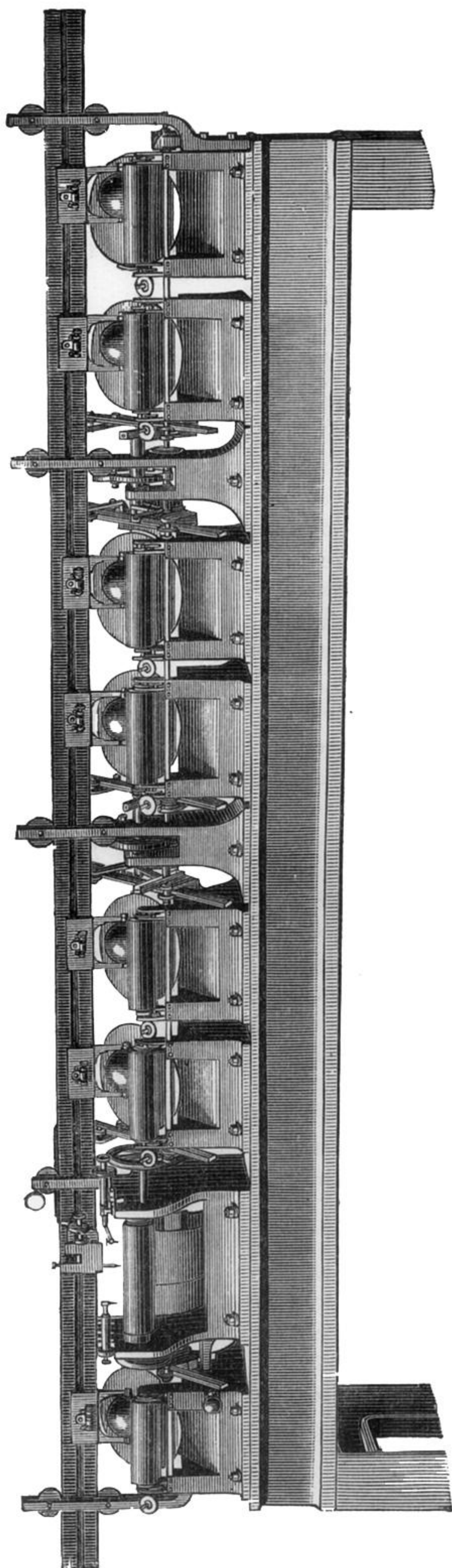


FIG. 2.

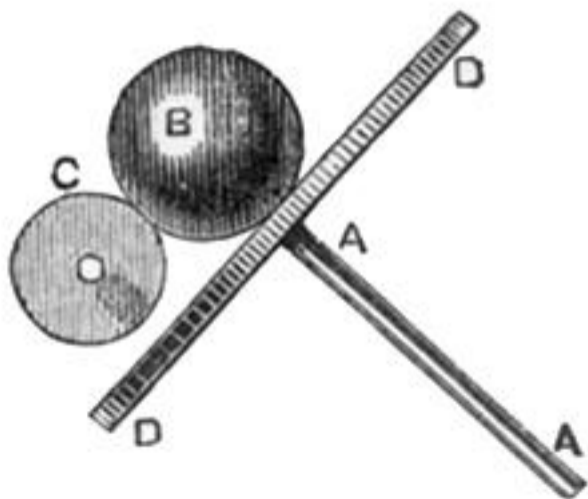


FIG. 3.

